20 October 2020 13:47

Q:→ Prove that the argument

$p \to \sim q$, $r \to q$, $r \vdash \sim p$

is valid without using truth tables.

Pf

1) $p \to \sim q$ (Premise)

2) $r \to q$ (Premise)

3) $\sim q \to \sim r$ (Contrapositive of 2) $[p \to q \equiv \sim q \to \sim p]$

4) $p \to \sim r$ (Hypothetical Syllogism of 1) and 3))

$$\begin{array}{l} p \to \sim q \\ \sim q \to \sim r \\ \hline \therefore p \to \sim r \end{array}$$

5) r (Premise)

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

6) $\sim p$ (Modus tollen of 4) and 5))

$$\begin{array}{l} p \to \underline{\sim r} \\ r \\ \hline \therefore \sim p \end{array}$$

$\therefore$ Given argument is valid

Q:→ Consider the following statements

It is snowing. If it is warm, then it is not snowing. If it is not warm then I cannot go for swimming.

Show that the statement "I cannot go for swimming" is a true statement."

sol; Let p: It is snowing

q: It is warm

r: I can go for swimming

Premises P_1 : p

P_2 : $q \to \sim p$

P_3 : $\sim q \to \sim r$

Conclusion Q : $\sim r$

Argument $p, q \to \sim p, \sim q \to \sim r \vdash \sim r$

1) p Premise

2) $q \to \sim p$ Premise

3) $\sim q \to \sim r$ Premise

4) $p \to \sim q$ (Contrapositive of 2))

$p \to q \equiv \sim q \to \sim p$

$q \to \sim p \equiv \sim(\sim p) \to \sim q$

$\equiv p \to \sim q$

5) $p \to \sim r$ (Hypothetical Syllogism of 4) and 3))

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \quad \text{i.e} \quad \begin{array}{l} p \to \sim q \\ \sim q \to \sim r \\ \hline \therefore p \to \sim r \end{array}$$

6) $\sim r$ (Modus tollen of 1) and 5))

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array} \quad \text{i.e} \quad \begin{array}{l} p \\ p \to \sim r \\ \hline \sim r \end{array}$$

$\therefore$ Argument is valid

9) Universal Instantiation

Premise $\forall x\, P(x)$

Conclusion $P(c)$ for any c

10) Universal Generalization

Premise $P(c)$ for any arbitrary c

Conclusion $\forall x\, P(x)$

11) Existential Instantiation

Premise $\exists x\, P(x)$

Conclusion $P(c)$ for some c

12) Existential Generalization

Premise $P(c)$ for some c

Conclusion $\exists x\, P(x)$

Q:→ It is known that

1 A student in this class not read the book.

2. Everyone in this class passed the first exam. Can you conclude that "someone who passed the first exam has not read the book."

sol :→ Let

$C(x)$ represents x is student in the class

$B(x)$ " " read the book

$p(x)$ " x passed the first exam

Premises P_1 : $\exists x\,(C(x) \wedge \sim B(x))$

P_2 : $\forall x\,(C(x) \rightarrow P(x))$

Conclusion $\exists x\,(P(x) \wedge \sim B(x))$

1) $\exists x\,(C(x) \wedge \sim B(x))$ Premise

2) $C(a) \wedge \sim B(a)$ (Existential Instantiation of 1) for some a)

3) $C(a)$ (Simplification of 2) $\dfrac{p \wedge q}{\therefore p}$

4) $\forall x\,(C(x) \rightarrow P(x))$ Premise

5) $C(a) \rightarrow P(a)$ (Universal Instantiation of 4)

6) $P(a)$ (Modus Ponen of 3) and 5) $\dfrac{\begin{array}{c} p \\ p \rightarrow q \end{array}}{\therefore q}$ $\dfrac{\begin{array}{c} C(a) \\ C(a) \rightarrow P(a) \end{array}}{\therefore P(a)}$

7) $\sim B(a)$ (Simplification of 2) $p \wedge q \equiv \dfrac{q \wedge p}{\therefore q}$

8) $P(a) \wedge \sim B(a)$ (Conjuction of 6) and 7)) $\dfrac{\begin{array}{c} p \\ q \end{array}}{\therefore p \wedge q}$ $C(a) \wedge \sim B(a) \equiv \dfrac{\sim B(a) \wedge C(a)}{\therefore \sim B(a)}$

9) $\exists x\,(P(x) \wedge \sim B(x))$ (Existential Generalization)